South Florida Water Management District

Control Concepts for Storm Water Treatment Areas

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SUMMARY

CONTROL CONCEPTS FOR STORM WATER TREATMENT AREAS

This report summarizes the system identification method proposed to identify a realistic model for the water level control of the flow through storm water treatment areas. In section I an introduction to the basic tenets of system identification is given followed by a system model identified with available data outlined in section II along with a controller design in section III. The success of the system ID process will depend on the ability to collect water level data or phosphorus data over a frequency range. This can be accomplished by changing the input variables in a square wave pattern and observing the levels at chosen locations as a function of time. Since such data is not available currently the presented work simply provides some insight into what can be achieved by carrying out a well-planned experiment for the purpose of system ID. The trends shown are representative of what can be expected. These ideas will be illustrated at the workshop to explain how the control system can be designed and implemented first on an experimental facility and then as a pilot test.

I. INTRODUCTION TO SYSTEM IDENTIFICATION

System identification is a process by which a predictive mathematical description for a physical system is obtained from real physical data. In its simplest form, input-output data are related by a set of differential equations relating each input variable to an output variable. There are some guidelines that must be followed in selecting the input excitations so that the system ID leads to a realistic model. By doing some simple experiments the necessary input functions can be selected based on any available analytical models, expert opinions on the subject and available bandwidth information.

The system identification is important for the design and implementation of a control system. The confidence level of the validity of a system model dictates the controller design process. The fidelity of the model realized will inform the controller designer what methodology to use. An identification process could lead to a state space model or transfer matrix model, possibly nonlinear. For purposes of control design one uses the simplest possible dynamic model that is able to capture the general input-output behavior. Every system identification process consists of the following stages:

- Experimenting and data collection
- Signal conditioning and data processing
- Model structure selection
- Parameter estimation
- Model validation.

Experiment and data collection is an important step of an identification process without which a model cannot be built. Based on prior knowledge of the system and its expected or desired performance the input excitations must be chosen. There are two very important considerations:

In its simplest way, the system model may be treated as linear time invariant. To identify the parameters of a linear system we need to first determine its model order and the relative degree. This is usually done by iterating over the order of the dynamic system. The primary requirement is that the input should satisfy the persistent excitation condition. That means that the input should be chosen to excite all the relevant dynamics of the system. In other words, the frequency content of the input signal should be rich enough to be able to identify all the necessary parameters of the linear system.

However, a linear system is an idealization of the real world. All real systems need one or more nonlinear elements for their model development. The identification process for that is to assume a parametric form of the nonlinear elements in a system and estimate the parameters, and then use a nonlinear set of differential equations to describe the system. While it is useful to be able to obtain such nonlinear models in control design in most cases one can work profitably with linear models and compensate for any uncertainty with robustness measures. The key idea is to obtain a cluster of linear models to represent the overall nonlinear behavior of the system. This is accomplished by linearizing the nonlinear system about an operating point. The error between linear estimated system and the actual nonlinear one would naturally increase as one moves away from the operating point and the system so identified. By identifying several linear systems the deviations from the actual nonlinear system can be kept to a minimum. The goal should be to obtain a minimum number of such linear models.

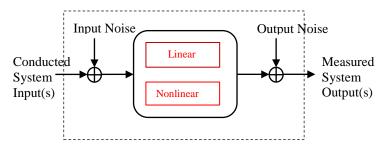


Figure 1: Real system schematic

In a real experiment, the output measured is influenced by sensor noise and input disturbances. Generated input stimuli can also be noisy. The collected data can be filtered using signal processing techniques before feeding them into the system identification module. The model structure of the physical system is dictated by the data we get from the system, and will influence the control strategy to be used. A typical model of the system takes the form of a Transfer Function, or a differential/difference equation relating an input-output pair. Additional measurements of signals and/or parameters in the interior of the system can be used to further refine a basic input-output model.

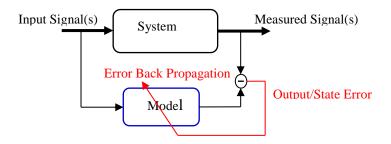


Figure 2: System Identification

Once a preliminary model and input-output data sets are chosen the specific parameters for the model order chosen can be estimated. The objective is to estimate the parameters so that the output error between the model output and experimental data is minimized. This could be done off-line with available input and output data with the model updated with new data when they become available. This process is repeated until the error achieved is less than a certain threshold. This can also be done on-line where at every sample time the error is back propagated through the model to modify the model parameters.

After parameter estimation, the model should be validated using a validation data set. It is done to avoid over training – or over estimating of the parameters of the system. Once the model is estimated using available data sets it validation should be done using a new set of data that was not previously used in the model identification. [1]**Error! Reference source not found.**

II. STORM WATER TREATMENT AREA

This project deals with model development for flow of water in Storm water Treatment Areas (STAs) in the Florida Everglades. STAs are huge wetland areas that have been constructed to reduce the level of chemicals such as phosphates in the water discharged into surface water in Florida. As water flows though the wetland, it is expected that the larger particles of chemical substances will deposit while the smaller particles are absorbed by plantation in the wetland. Both these phenomena are time dependent, and to achieve a desired level of chemicals in the water it has been postulated that it is necessary to control the water level in the wetland, and the resident time of the water flow.

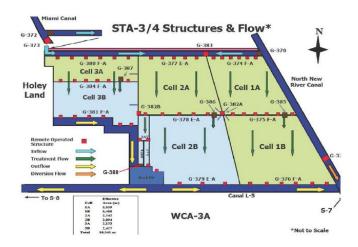


Figure 3: Canals and structures used to route water into the wetland STA-3/4 Cell 3A

The water level inside the wetland is controlled through inflow canals upstream, and outflow gates downstream of the wetland. The objective is to control the water level and also the water flow rates so that water has sufficient resident time in the wetland to be treated. STA-3/4 Cell 3 was selected as a pilot area to perform extensive observation.

In Cell 3, water gauges are situated to measure the water level at different points of the wetland. Since we do not have any information about what is happening in other parts of the system other than at the discrete points at which gauges were located, approaching from a partial differential equation model is not very practical although the availability of such a comprehensive model would certainly reduce the cost of having to run extensive experiments to collect data. Instead, we model the behavior of the measured parameters, (the water levels at the gauge locations), in response to the water flows at the inlet and outlet of the wetland.

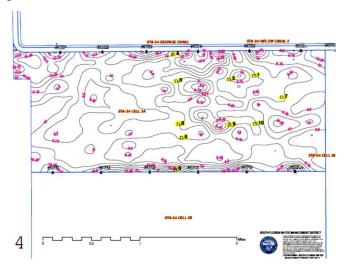


Figure 4: Contour plot of ground elevations in STA-3/4 Cell 3A, and gauge locations. Ten water level sensors were deployed (n1 through n10).

In particular, the water level measured with each gauge is modeled as a system of interconnected tanks. As the slope of the wetland is generally from north to south, water enters from the inflow canal into the wetland and flows down to the end. Taking this into account the gauges are clustered into three separate systems grouped as $\{n_4, n_3, n_8, n_5\}$, $\{n_4, n_2, n_9, n_5\}$, and $\{n_4, n_1, n_{10}, n_5\}$.

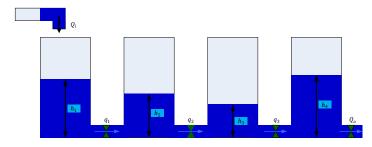


Figure 5: Four related tank model used to model the dynamics of the water level measured by the gauges in the wetland.

For a four-tank interconnected system, the following simple equations govern the water flow:

$$\begin{cases} q_1 = k_1(h_1 - h_2) \\ q_2 = k_2(h_2 - h_3) \\ q_3 = k_3(h_3 - h_4) \\ q_4 = Q_o \\ \begin{cases} a_1 \dot{h}_1 = -q_1 + Q_i \\ a_2 \dot{h}_2 = q_1 - q_2 \\ a_3 \dot{h}_3 = q_2 - q_3 \\ a_4 \dot{h}_4 = q_3 - Q_0 \end{cases} \Rightarrow \begin{cases} a_1 \dot{h}_1 = -k_1(h_1 - h_2) + Q_i \\ a_2 \dot{h}_2 = k_1(h_1 - h_2) - k_2(h_2 - h_3) \\ a_3 \dot{h}_3 = k_2(h_2 - h_3) - k_3(h_3 - h_4) \end{cases} ;$$

$$(1)$$

 Q_i and Q_o are the total water inflow and outflow of the wetland, q_i 's designate the water flow between regions, and k_i s are the coefficients indicating flow resistance at various points of the wetland, and a_i is the area of each region. The variation of the water level in each tank is a function of the difference in water level of the neighboring tanks at that instant. And the height in each tank is dependent on the resistance to flow from the vegetation. For example, one of the assumptions in deriving the above model is that the volume of the water in the system, considering the input and output water to it, is always conserved.

A model similar to the above analytical model can also be obtained using system identification using experimental data with the added advantage that some neglected dynamics may show up in the identified model. That would in turn help the development of a refined analytical model. To ascertain the validity of models an extensive identification experiment has been done on wetland STA -3/4 Cell 3A. The experiments included a sinusoidal discharge of water into the wetland from different levels characterizing the operating points. **Error! Reference source not found.** shows the water levels in the wetland that are registered with a set of gauges $\{n_4, n_2, n_9, n_5\}$. The slope in the wetland is such that there is a casual relationship between the water level in these gauges. The sinusoidal discharge wave appears in the data logged by the gauges with some delay. This experiment was done by perturbing the water levels – system states – around the

operating point of 3.1-3.5ft. Two other experiments have been done to model the behavior of the system in the range 2.8-3.2 ft, and 2.2-2.6 ft ranges of the water level inside the wetland. This allows the possibility of identifying a linear system that can be used to design a controller valid for each interval of operating conditions.

For the identification process to converge we need to eliminate the sudden small discontinuities in the data logged, and obtain a smooth data set. To do so, a first order digital low pass filter given below is designed to eliminate noise and jumps in water level:

$$H(z) = \frac{0.15}{z - (1 - 0.15)} \tag{2}$$

The filtered data are then used to identify the dynamics of the system; however, real data are used in simulations to compare the performance of the real system with the identified model.

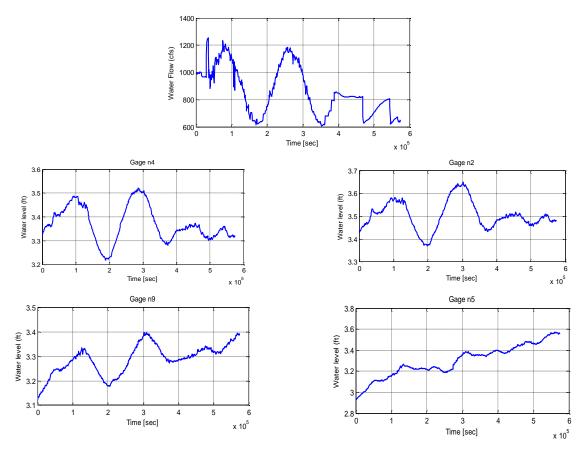


Figure 6: Total Discharge, and Water Level at the Gauges $\{n_4, n_2, n_9, n_5\}$.

A linear model was chosen with the water level in each gauge as output to describe the internal states of the system. It means that for a degree-four system we considered the output measurement matrix \mathcal{C} to be the identity matrix. It is possible to restrict the identification process to set degree, but in this case the result may not be satisfactory especially due to the absence of a good predictive analytical model. Consequently, in the proposed method the identification is carried out for a general state space model, and the system is transferred to the desired structure

using a similarity transform. The data sets available can be used to estimate a linear system for the wetland around each of the operating points at 3.3, 3, and 2.4 feet of water level in the wetland.

In the remainder of the report we study the behavior of the water level in gages $\{n_4, n_3, n_8, n_5\}$ for the first test wave, about the operating point 3.3ft. The goal is to identify a dynamic system that relates the water level in these gauges to the input flow. With the model structure and water level in each gauge in hand we can construct a state space model for the wetland. The mathematical background needed to understand the identification procedure can be obtained from references to the subject, such as [1], **Error! Reference source not found.**, and **Error! Reference source not found.**. The derived state space model for the system of gages $\{n_4, n_3, n_8, n_5\}$ around the operating point is,

$$X[k+1] = AX[k] + BU[k]$$

$$A = \begin{bmatrix} 0.925 & 0.057 & 0.012 & 0.003 \\ -0.023 & 1.012 & 0.001 & -0.006 \\ 0.046 & -0.025 & 0.9584 & 0.015 \\ -0.055 & 0.074 & -0.014 & 0.996 \end{bmatrix}, B = \begin{bmatrix} 2.609e - 5 \\ 1.652e - 5 \\ -6.683e - 7 \\ 6.132e - 6 \end{bmatrix}$$
(3)

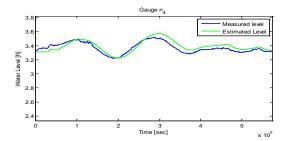
X: vector of the water levels measured

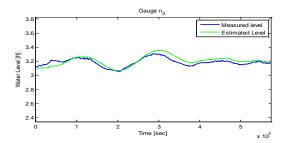
U: Inflow water

T=900s: Sampling Time

The state space equations of the identified system tell us about the dynamics of the system. In particular one could assess the coupling between various sub systems and the controllability from the chosen input variable.

Figure 7 shows the measured data and the estimated water level in each gauge using the identified model. As seen the identified dynamic system has promising performance that could be conveniently used in controller design. The same identification process can be used to get dynamic system models that relate the gauges (states) at different operating points.





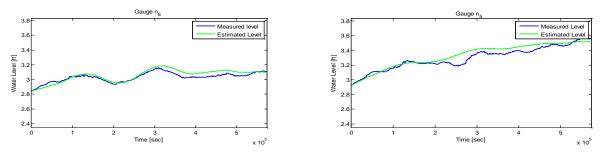


Figure 7: Estimated and Measured Water Level in Gauges $\{n_4, n_3, n_8, n_5\}$.

III. WATER LEVEL CONTROL INSIDE THE WETLAND AREA

Once a model of the system behavior of the water level in wetland is identified from the experimental data sets, this model is used to design a controller. The control objective in this case was chosen as:

- Water level regulation, and
- Disturbance rejection

The output variables to be controlled are the water levels inside the wetland area, measured with two middle gauges (n_2 and n_9). The data set consists of one control input to the system, which is the total inflow to the wetland. With a single input, only the water level in one of the gauges is controllable. To deal with a more realistic scenario an auxiliary input is added to the system. This second input is taken as the outflow from the wetland at downstream. Now, they may be used to control the water level in two middle gauges which are the best measures of the water level inside the wetland area. Figure 8 is a schematic of the control structure.

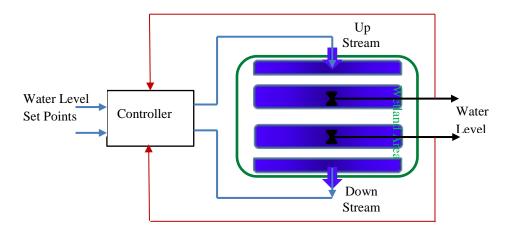


Figure 8: Control Structure

The variation of the water level measured by n_5 is assumed proportional to the downstream water discharge. It is important to mention here that the obtained model is based on the water flows inside/outside of the wetland. To have an applicable design the actuators model, the model

of the water discharge equipment including the gates and the pump, also should be included in control loop, and considered in design stage.

$$X[k+1] = AX[k] + BU[k]$$

$$Y[k] = CX[k]$$

$$A = \begin{bmatrix} 0.925 & 0.057 & 0.012 & 0.003 \\ -0.023 & 1.012 & 0.001 & -0.006 \\ 0.046 & -0.025 & 0.9584 & 0.015 \\ -0.055 & 0.074 & -0.014 & 0.996 \end{bmatrix}, B = \begin{bmatrix} 2.609e - 5 & 0 \\ 1.652e - 5 & 0 \\ -6.683e - 7 & 0 \\ 6.132e - 6 & -1e - 4 \end{bmatrix}$$

$$C = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

X: vector of the water levels measured

 $U: \begin{bmatrix} Inflow \\ Outflow \end{bmatrix}$

T=900s: Sampling Time

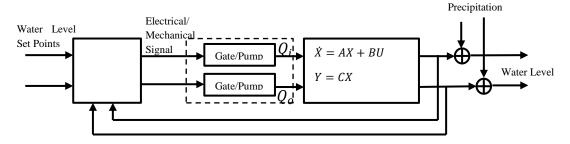


Figure 9: Closed Loop Control Structure

For simplicity, the dynamic of the actuators are neglected in this study.

The design objective is to regulate the water level in spite of any precipitations. The water level in the wetland should remain in a certain ranges due to several reasons: The water needs to flow in the wetland so that the undesirable chemicals deposit and are separated; the necessary vegetation in the wetland to absorb and alter the undesirable particles and substances can get damaged in a flooded wetland; and the wetland is a natural habitat of several protected species, and needs to be preserved.

Following is the Controllability Gramians of the system:

$$\Sigma_C = \begin{bmatrix} 0.038 & 0.041 & 0.008 & 0.028 \\ 0.041 & 0.061 & -0.018 & -0.015 \\ 0.008 & -0.018 & 0.052 & 0.087 \\ 0.028 & -0.015 & 0.087 & 0.324 \end{bmatrix}$$
(5)

Controllability Gramians of the system make it clear that although there is a large interaction between the states of the system, the input set is able influence the desired outputs. Since the controllability is assured with the Gramians being non-singular we could close the loops around the system separately, and do a fine tuning later. To do so, a PID-type controller is used to design each control loop. First, a PID controller is designed for the loop of first input-output to stabilize the system and reject the disturbance. Then the second controller is designed, considering the first one in the loop. Then a fine tuning is done to achieve the final required performance.

IV. SIMULATION SCENARIO

The control objective is to regulate the water level in the wetland despite precipitation. The desired performance (design) specifications can be chosen as minimum overshoot and rise time, along with reasonable amount of control efforts – inflow and outflow water discharge rate.

The unique control constraint in this problem is that the second control signal can only be positive. It means that it has just out-flowed from the downstream gate. Then in the controller design the second input signal could just be positive. Also, there are bounds on the input and output signals, which although is not exceeded in this design, must be considered in future work.

For the simulation scenario, the initial level of water is assumed 3ft in each gauge. The control objective is to set the water level at the gauges n_3 and n_8 at 2.8ft and 3.2ft, respectively. Also there would be 24 hours of precipitation at the rate of 1e-7 (cft/s)/ft².

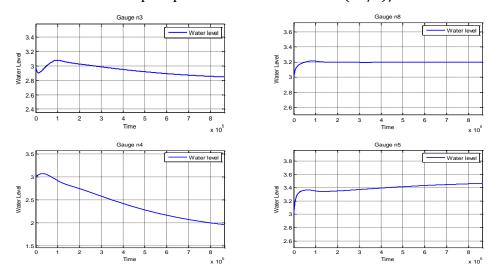


Figure 10: Water level read in each gauge during the simulation

Figure 10 is the water level changes at each gauge. As it is clear, the designed controller shows acceptable performance.

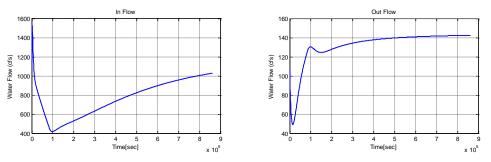


Figure 11: The water flow in and out of the wetland during the simulation

Also, to have an implementable control design, it is important that our control signals, here the in/out flow water discharges, be in a certain ranges. Compared to the experimental data of **Error! Reference source not found.**, Figure 11 shows promising results.

Appendix A

Software Package

Appendix B

St. Venant equations developed for analyzing depth averaged, shallow, and one-dimensional canal flow, can be adopted to use in a wetland. These equations consist of two equations of continuity and momentum.

$$\frac{\partial h}{\partial t} + \frac{\partial q}{\partial x} = 0$$

$$\frac{\partial q}{\partial t} + \frac{\partial}{\partial x} \left(\frac{q^2}{h} \right) + gh \left(s_f - s_0 + \frac{\partial h}{\partial x} \right) = 0$$
(6)

where

q: discharge per unit width,

h: water depth,

q = uh,

u: average flow velocity,

g = 9.81: gravitational acceleration,

 s_f : friction slope,

 s_0 : River bed slope.

However, formulation for water flow in a canal with vegetation resistance is more complicated, and generally couldn't be done with the simple equations of *St. Venant*. This difficulty is rooted in several facts, such as,

- Vegetation resistance depends on the bottom topography which might not be flat or even stationary due to organic accretion, sedimentation and erosion.
- Vegetation resistance depends on vegetation density and type that varies spatially, and are not quantifiable easily.
- Flow may be locally two dimensional in certain areas.

Many models have been proposed to suit various flow conditions. *Manning's* equation is the most common model to describe flow resistance in depth averaged canals. Following simple power function law is a variant of the Manning's equations used by [4] for a variety of wetland types.

$$q(h, s_f) = \frac{1}{n_h} h^{1+\gamma} |s_f|^{\alpha} \operatorname{sgn}(s_f)$$
(7)

The three parameters, n_b , γ , and α determine three basic behaviors of the flow within a wetland. When $\gamma = 2/3$ and $\alpha = 1/2$, n_b is the Manning roughness coefficient and the equation becomes the Manning's standard equation applicable for free surface depth averaged flow. There is no guarantee that the vegetation roughness always follows the above equation; however, this equation is simple based on power functions whose exponents provide valuable physical meanings.

The Manning formula is an empirical formula for estimating open channel flow or free-surface flow driven by gravity, and the vegetation roughness is an important physical parameter to model the Storm Water Treatment Areas (STAs) and Flow Equalization Basins (FEBs). Manning's roughness values have been used for STA designs at SFWMD in the past. These values calculated using data in DBHYDRO for upstream and downstream could be considered as bulk resistance parameters, and are useful for a limited number of applications. Management of STAs for optimizing phosphorous removal requires more detailed estimation of the vegetation resistance values.

Using the St. Venant equations, a simplified linear equation could be found governing the water level in a one dimensional canal:

$$\frac{\partial h}{\partial t} + a(h, s_f) \frac{\partial h}{\partial x} = K(h, s_f) \frac{\partial^2 h}{\partial x^{2'}}$$
(8)

where a is defined as the kinematic celerity or propagation speed and K is hydraulic diffusivity or attenuation rate. These are two characteristics of a linear hyperbolic-parabolic partial differential equation governing convective-diffusive phenomena. Comparing equation (8) to the St. Venant equation, these parameters could be defined as follows:

$$a(h, s_f) \triangleq \frac{\partial q}{\partial h}, \qquad K(h, s_f) \triangleq \frac{\partial q}{\partial s_H}.$$
 (9)

When the depth is shallow and the slope is large the term with a dominates, and the equation becomes predominantly hyperbolic. On the other hand, when the canal is deep and the slope is small the term with K dominates, and the equation becomes predominantly parabolic. These parameters also could be related to the Manning parameters and roughness coefficient as following:

$$a(h, s_f) \triangleq \frac{\partial q}{\partial h} = (1 + \gamma) \frac{1}{n_b} h^{\gamma} |s_f|^{\alpha} \operatorname{sgn}(s_f) = (1 + \gamma) \frac{q}{h}$$
(10)

$$K\big(h,s_f\big)\triangleq\frac{\partial q}{\partial s_H}=\frac{\partial q}{\partial s_f}=\frac{\alpha}{n_b}h^{1+\gamma}\big|s_f\big|^{\alpha-1}=\alpha\frac{q}{s_f}$$

The advantage of the power function form of the vegetation roughness equation is to relate the three basic properties of wave speed, attenuation and discharge to three parameters of a, K, and q in the linearized hyperbolic-parabolic equation. To identify these parameters a sinusoidal solution is applied on the governing equation (8) using the complex form $h = c.e^{ft-kx}$ where f and k are both complex numbers. Choosing $= f_2i$, f_2 would be the frequency of the discharged wave introduced at the upstream; then, $k = k_1 + k_2i$ determines the spatial decay constant and wave length. The real part of the solution gives the physical solution for the system:

$$h = c.e^{-k_1 x}.\cos(f_2 t - k_2 x) \tag{11}$$

The values of the K, and α are to be calculated using

$$a(h, s_f) \triangleq \frac{\partial q}{\partial h} = \frac{f_2}{k_2} \frac{(k_2^2 - k_1^2)}{(k_2^2 + k_1^2)}$$
 (12)

$$K(h, s_f) \triangleq \frac{\partial q}{\partial s_H} = \frac{f_2}{k_2} \frac{k_1}{(k_2^2 + k_1^2)}$$

In a fully diffusive system, a=0, and $k_1=k_2$. In a fully hyperbolic system K=0, $k_1=0$, and $k_2=\frac{f_2}{C}$, where C is the wave speed. In all cases $k_1< k_2$, unless the system has a source term. This equation shows that both K and A can be derived from the wave speed and attenuation.

Although the partial differential equation (8) gives us a very good sense about the governing regime of the wetland, it is difficult to use it as a model to control the system. Using these equations needs to have complete picture of the interactions inside the wetland. When the wetland vegetation is very thick at the root, or when there is a muck layer with dead plants at the bottom of canal, there can be multiple flow regimes at different depths of a wetland. Also, uneven ground elevation can create a flow cutoff depth below which the discharge is too small to measure. All these makes the differential equation related to the position parameter x useless in control purpose. However, all these analysis gives us a broad vision about the phenomena taking place in a wetland.

To identify the bulk parameters of the wetland, water is pumped at the upstream end to create waves. The upstream end has a spreader canal which is used to distribute the water evenly across the wetland. The storage effect of the canal at the upstream end of the wetland causes attenuation in the amplitude and a lag in the water level wave.

The equation written for steady flow in a wetland of width *B* connected to a canal of the same length is as follow:

$$A_c \frac{\partial H_c}{\partial t} = Q_i - \left[\frac{1}{n_b} B h_0^{1+\gamma} \left(\frac{\partial h_0}{\partial x} \right)^{\alpha} \right]_{r=0}$$
 (13)

 A_c : Plan area of the canal;

B: Width of the wetland equal to the length of the canal;

 H_c : Steady state water level in the canal;

 h_0 : Water level in wetland;

 Q_i : Uniform rate of discharge into the canal;

If the steady discharge of Q_i is disturbed by a sinusoidal rate of $q_0 \sin(f_2 t)$, the disturbed equation will be

$$A_{c} \frac{\partial (H_{c} + h_{c})}{\partial t} = Q_{i} + q_{0} \sin(f_{2}t) - \left[\frac{1}{n_{b}} B(h_{0} + \hat{h})^{1+\gamma} \left(\frac{\partial (h_{0} + \hat{h})}{\partial x} \right)^{\alpha} \right]_{x=0}$$

$$(14)$$

Doing the calculations, the time lag between the waves cross the canal and penetrate to the wetland and the discharge rate is obtained as

$$\beta = \tan^{-1} \left(\frac{B_c f_2 + K k_2}{a + K k_1} \right) \tag{15}$$

 B_c is the width of the canal, f_2 is the frequency of the discharged waves, k_2 is the spatial decay constant, and k_1 is the wave length. The time lag is proportional to the disturbing frequency and the canal width. For kinematic flow when K = 0 ($f_2 = 0$), the phase lag will be zero, and for a fully diffusive flow where a = 0 the phase lag is 90 degrees.

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